

Contents

Acknowledgement	i
Abstract	iii
Publication List	viii
1 Introduction	1
1.1 Topology in magnets	1
1.1.1 One dimension	3
1.1.2 Two dimensions	4
1.1.3 Three dimensions	6
1.2 Topological spin crystal	8
1.2.1 Chiral soliton lattice	9
1.2.2 Skyrmion lattice	11
1.2.3 Hedgehog lattice	12
1.3 Emergent electromagnetism	14
1.3.1 Emergent magnetic field	16
1.3.2 Emergent electric field	17
1.3.3 Emergent electromagnetic phenomena	18
1.4 Experiments on topological spin textures	20
1.4.1 Chiral soliton lattice	20
1.4.2 Skyrmion lattice	22
1.4.3 Hedgehog lattice	24
1.5 Stabilization mechanism	27
1.5.1 Dzyaloshinskii-Moriya interaction	27
1.5.2 Frustrated exchange interactions	28
1.5.3 Itinerant frustration	29
1.6 Purpose of this thesis	31
1.7 Organization of this thesis	32

2	Spin moiré engineering	33
2.1	Moiré pattern	33
2.2	Spin moiré and its engineering	36
3	Model	40
3.1	Phenomenological model	40
3.2	Spin model	41
3.3	Spin-charge coupled model	43
3.4	Effective spin model	45
3.4.1	Bilinear effective spin model	45
3.4.2	Bilinear-biquadratic effective spin model	49
4	Methods	52
4.1	Variational calculation	52
4.2	Energy minimization	53
4.3	Extracting moiré parameters	54
4.4	Landau-Lifshitz-Gilbert equation	54
4.5	Linear spin-wave theory	56
4.6	Phase degree of freedom and hyperspace representation	57
4.6.1	Phase degree of freedom	57
4.6.2	Hyperspace representation	58
5	Spin moiré engineering in one dimension	61
5.1	Magnetic resonance and soliton penetration	62
5.1.1	Ground-state phase diagram	62
5.1.2	Linear spin-wave analysis	64
5.1.3	Edge excitation and soliton penetration	67
5.2	Emergent electric field from magnetic resonance	72
5.2.1	Physical quantities	73
5.2.2	Bulk properties	74
5.2.3	Edge contributions	83
5.2.4	Order estimate	89
5.3	Driving spin textures	90
5.3.1	Physical quantities	90
5.3.2	$h_x^{\text{stat}} = 0$ case	92
5.3.3	$h_x^{\text{stat}} > 0$ case	94
5.4	Concluding remarks of this chapter	98
6	Spin moiré engineering in two dimensions	99
6.1	Phase shift in $3Q$ -SkL	99
6.1.1	Hyperspace representation of the $3Q$ states	100

6.1.2	Screw $3Q$ state	102
6.1.3	Sinusoidal $3Q$ state	112
6.1.4	Numerical analysis of phase shift in the sinusoidal $3Q$ state	119
6.2	Crystallization dynamics of $2Q$ -SkL	123
6.2.1	Ground-state phase diagram	124
6.2.2	Spin structure factor in the coarsening process	126
6.2.3	Orientational and translational orders in the SkL	129
6.2.4	Dislocations in the SkL	131
6.2.5	Emergent XY model	134
6.3	Concluding remarks of this chapter	137
7	Spin moiré engineering in three dimensions	139
7.1	Twist angle control in $3Q$ -HL	139
7.1.1	Spin configurations	141
7.1.2	Topological properties	144
7.1.3	Topological phase diagram	148
7.1.4	Variational calculation	151
7.2	Phase shift in $4Q$ -HL	157
7.2.1	Hyperspace representation of the $4Q$ states	159
7.2.2	Screw $4Q$ state	160
7.2.3	Sinusoidal $4Q$ state	171
7.2.4	Numerical analysis of phase shifts in the screw $4Q$ state	179
7.3	Current-induced dynamics of $3Q$ -HL	181
7.3.1	Phase diagram	183
7.3.2	Longitudinal and transverse motions of hedgehog	187
7.3.3	Potential barrier from lattice discretization	192
7.4	Concluding remarks of this chapter	197
8	Summary	199
A	Soliton penetration from chirality domain boundary	205
B	Surface barrier	208
C	Some analytical formulae for $\chi_{E,\mu}(\omega)$	211
C.1	$\chi_{E,\mu}(\omega)$ on resonance	211
C.2	$\chi_{E,\mu}(\omega)$ in large- ω region	212
D	Counting dislocations	214