Abstract

Electrons in solids possess multiple degrees of freedom, including charge, spin, and orbital components, which can give rise to a wide variety of quantum phases and emergent phenomena. In condensed matter physics, attention is often focused on the low-energy properties of such systems, where interactions among these degrees of freedom describe the essential physics. As a result, multiple ordering tendencies frequently compete at low energies, leading to complex phase diagrams and unconventional behaviors. Thermal and quantum fluctuations further enhance this competition, making quantum materials a fertile platform for rich and sometimes unexpected phenomena.

Since the late twentieth century, the concept of competing phases has been extensively explored in condensed matter physics as a key mechanism underlying unconventional phenomena in quantum materials. Heavy fermion systems, colossal magnetoresistance manganites, and cuprate high-temperature superconductors have been established as paradigmatic examples, in which the competition among multiple low-energy phases, tuned by pressure, magnetic field, or chemical composition, gives rise to rich phase diagrams and anomalous physical properties. These studies have demonstrated that the delicate balance among competing orders, amplified by thermal and quantum fluctuations, plays a decisive role in shaping the low-energy physics of correlated electron systems. More recently, advances in the understanding of topological phases have further expanded this framework by revealing that quantum phases can be distinguished not only by symmetry-breaking orders but also by global entanglement and topological properties of the many-body wave function. Thus, the interplay between topological phases and conventional symmetry-breaking orders has emerged as a new frontier in condensed matter physics.

In this thesis, we explore quantum phenomena arising from the competition between distinct quantum phases. Our study is twofold: one part focuses on localized spin systems, and the other on itinerant electron systems. In the first part, we address quantum spin systems with spins larger than $S = 1/2$. Specifically, we investigate two distinct $S = 3/2$ models on two-dimensional lattices. The first model incorporates bilinear, biquadratic, and bicubic interactions on square and triangular lattices, where dipole, quadrupole, and octupole degrees of freedom compete with each other. Analyzing the ground-state phase diagrams using semi-classical and quantum approaches, we elucidate how quantum phases emerge from the competition among different interactions. The second model interpolates between the Kitaev model and the Affleck-Kennedy-Lieb-Tasaki model on the honeycomb lattice. Using classical, semi-classical and quantum approaches, we reveal a rich ground-state phase diagram. In particular, we show that noncoplanar orders observed in the classical and semi-classical analyses are progressively suppressed as quantum fluctuations are incorporated, suggesting the emergence of a quantum-entangled state in the competing regime between two distinct quantum disordered states, the Kitaev quantum spin liquid and the Affleck-Kennedy-Lieb-Tasaki valence bond solid. These studies shed light on the rich physics of higher-spin systems, where different magnetic orders and quantum spin liquid states compete due to frustration and quantum fluctuations.

In the second part, we turn to itinerant electron systems on two-dimensional lattice structures. Employing the Haldane model as a prototypical example of a Chern insulator, we investigate how topological phases compete on two distinct lattice geometries by connecting them via lattice geometry or transfer integrals. First, we study the Haldane model on a 1/6-depleted honeycomb lattice, and investigate the topological phase diagram across a wide range of model parameters. We also explore a connection between this lattice and the original honeycomb lattice by continuously changing the transfer integrals, revealing the emergence of topologically nontrivial insulating phases with high Chern number $C = \pm 2$ and metal-insulator transitions. Second, we consider a honeycomb lattice structure with selective random defects in a structurally controlled manner. By introducing a parameter that quantifies the defect concentration, we continuously interpolate between the pristine honeycomb lattice and its 1/6-depleted lattice, and investigate the spectral and topological properties of the Haldane model on these structures. We unveil that selective random defects induce a topological transition, which can be effectively captured by an effective model where defects are effectively represented as modulations of hopping amplitudes.

This thesis thus provides insights into quantum phenomena arising from competition between distinct quantum phases in both localized spin systems and itinerant electron systems. Our findings contribute to the broader understanding of competing orders in quantum materials, highlighting the rich physics that emerges from the interplay of interactions, quantum fluctuations, and topology.